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TOLERANCES ON ENERGY DEVIATION IN MICROBUNCHED ELECTRON COOLING*

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Abstract

The performance of microbunched electron cooling (MBEC) [1] is highly dependent on the quality of the hadron and cooler electron beams. As a result, understanding the influence of beam imperfections is very important from the point of view of determining the tolerances of MBEC. In this work, we incorporate a non-zero average energy offset into our 1D formalism [2, 3], which allows us to study the impact of effects such as correlated energy spread (chirp). In particular, we use our analytical theory to calculate the cooling rate loss due to the electron beam chirp and discuss ways to minimize the influence of this effect on MBEC.

INTRODUCTION

In MBEC, the hadron beam imprints an energy modulation on the co-propagating cooler electron beam in the modulator section of the machine. This energy modulation is then converted into a density modulation (bunching) after the e-beam passes through a dispersive chicane with strength $R_{56}^{(e,1)}$ (Fig. 1). In the meantime, the hadrons go through a separate section of the lattice, which also includes a chicane with strength $R_{56}^{(h)}$. The bunched electron beam then once again interacts with the hadrons in the kicker section, in a way that can ultimately lead to a significant reduction in the hadron energy spread (cooling of the transverse emittance is also possible but, for simplicity, we neglect this effect in this work). In order to accelerate this process and ensure that the cooling timescale is small enough for practical purposes, additional amplification stages are typically required, in which the bunching of the electron beam is boosted through plasma oscillations. Each such plasma stage consists of a drift space followed by a chicane of strength $R_{56}^{(e,j)}$ ($j = 2, ..., M+1$, where M is the total number of stages). For simplicity, we will assume that all stages have the same length L_d. In Refs. [2, 3] we derived the cooling timescale using a technique that tracks the microscopic fluctuations in the hadron and electron beams. The main results can be summarized as follows: the characteristic cooling time for the energy spread N_c — normalized by the ring revolution period T — is given by $1/N_c = A_0 I$, where

$$A_0 = \frac{4 I_e L_m L_k r_h}{\Sigma^3 \pi \gamma^3 I_A \sigma_e \sigma_h} \times \left(\frac{1}{\sigma_e} \sqrt{\frac{2 I_e}{\gamma I_A}} \right)^M \tag{1}$$

is a pre-factor and the cooling integral I is expressed by

$$I = (-1)^M \times 2 q_h q_{e,1} q_{e,2} ... q_{e,M+1} \int_0^\infty d\varkappa \varkappa^2 H^2(\varkappa)$$
$$\times \exp(-\varkappa^2 (q_h^2 + q_{e,1}^2 + q_{e,2}^2 + ... + q_{e,M+1}^2)/2)$$
$$\times \left(\frac{\varkappa H(r_p \varkappa)}{r_p} \right)^{M/2} \sin^M \left(r_p \frac{\Omega_p L_d}{c} \sqrt{\frac{2\varkappa H(r_p \varkappa)}{r_p}}\right). \tag{2}$$

In the expressions given above, γ is the relativistic factor (common for the co-propagating hadron/electron beams), L_m and L_k are the lengths of the modulator and kicker sections, $r_h = (Ze)^2/m_h c^2$ is the classical radius of the hadrons, I_e is the electron beam current and $I_A = m_e c^3/e \approx 17$ kA is the Alfven current. Moreover, σ_h and σ_e are, respectively, the rms energy spread values for the hadron and electron beams (assuming a Gaussian energy distribution for both). As far as the transverse properties of the beams are concerned, we again adopt Gaussian profiles and assume that a) at the modulator and kicker, the interacting beams have an identical, circular cross section characterized by an rms size Σ b) at the plasma stages, the e-beam is also round but with a different rms size $r_p \Sigma$, where r_p is a dimensionless squeeze factor. The latter quantity is also involved in the definition of the plasma oscillation frequency Ω_p, which is given by $\Omega_p = (c/r_p \Sigma)(I_e/\gamma^3 I_A)^{1/2}$. In Eq. (2), $q_h = R_{56}^{(h)} \sigma_h \gamma / \Sigma$ is the scaled hadron chicane strength and $q_{e,j} = R_{56}^{(e,j)} \sigma_e \gamma / \Sigma$ are the normalized strengths of the various electron chicanes. Lastly, the important function $H(\hat{k})$, which is directly related to the Fourier transform of the space charge interaction function, is defined by $H(\hat{k}) = \hat{k} \int_0^\infty d\tau \tau \exp(-\hat{k}^2 \tau^2/4)/(\tau^2 + 4)$.

ENERGY ERROR

In the derivation of Eq. (2), we have assumed a zero central value for the electron energy variable. In what follows, we discuss what would change if we were to remove this assumption. To begin with, we stipulate that the energy deviation of the electron beam, $\Delta\eta$, does not change the interaction between hadrons and electrons in the modulator and the kicker. It does, however, shift the wake generated by a hadron in the electron beam relative to the case when both beams have the same γ. If no plasma stages are present, the longitudinal shift

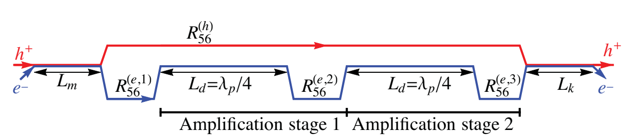

Figure 1: MBEC configuration with two plasma stages (the length L_d is a free parameter but, in practice, its value is $\sim \lambda_p$, where λ_p is the plasma wavelength).

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of the wake is equal to $\Delta z = R_{e,1}\Delta\eta$. For the dimensionless coordinate $\gamma\Delta z/\Sigma$ we have $\gamma\Delta z/\Sigma = q_{e,1}(\Delta\eta/\sigma_e) \equiv q_{e,1}\Delta p$, where we recall that $q_{e,1} = R_{56}^{(e,1)}\sigma_e\gamma/\Sigma$. The shift of the wake adds a phase factor to the space charge impedance $\mathcal{Z}(\varkappa)$, according to

$$\mathcal{Z}(\varkappa) \to \mathcal{Z}(\varkappa)e^{-i\varkappa q_{e,1}\Delta p}\,. \tag{3}$$

Hence, in our cooling calculations we should use this modified impedance, instead of the original one derived in [2]. Correspondingly, instead of Eq. (66) in the above-mentioned reference (or, equivalently, Eq. (2) for $M = 0$), we need to calculate the factor $I(q_h, q_{e,1}, \Delta p)$ given by

$$I(q_h, q_{e,1}, \Delta p) = 2q_{e,1}q_h \times \int_0^\infty d\varkappa\, \varkappa^2 e^{-\varkappa^2(q_{e,1}^2+q_h^2)/2} H^2(\varkappa)\cos(\varkappa q_{e,1}\Delta p). \tag{4}$$

The plot of this function — normalized by its value at the origin (no energy error) — for $q_{e,1} = q_h = 0.6$ is shown in Fig. 2.

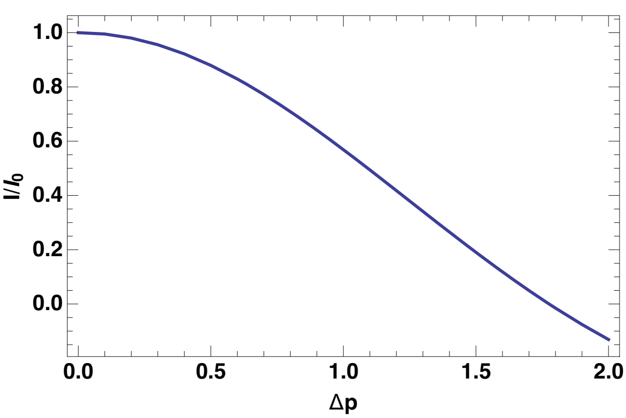

Figure 2: Plot of I/I_0 (I_0 is the value of I at the origin) versus Δp.

From this figure it follows that the relative energy deviation of $0.7\sigma_e$ leads to 23% loss of the cooling rate. This imposes a tolerance on the energy jitter of the electron beam.

LINEAR, QUADRATIC AND CUBIC ENERGY CHIRP

We now consider three different cases of correlated energy spread (chirp). If we have a linear chirp in the electron beam, i.e. $\Delta\eta(z) = h_1 z$ (where z is the position within the beam), we will characterize it with an rms value of the correlated energy variation, $\Delta\eta_{\rm rms}$. The latter is given by

$$\Delta\eta_{\rm rms}^2 = \int_{-\infty}^{\infty}(h_1 z)^2 F_e(z)dz = h_1^2\sigma_z^2, \tag{5}$$

where $F_e(z) = \exp(-z^2/2\sigma_z^2)/(\sqrt{2\pi}\sigma_z)$ is the longitudinal distribution function in the electron beam (normalized by unity) and σ_z is the rms electron bunch length. Thus, the chirp profile can be re-written in scaled units as

$$\Delta p(z) = \Delta p_{\rm rms}\frac{z}{\sigma_z}\,, \tag{6}$$

where $\Delta p_{\rm rms} = \Delta\eta_{\rm rms}/\sigma_e$. For the case of a quadratic chirp, we instead have $\Delta\eta(z) = h_2 z^2$, so that

$$\Delta\eta_{\rm rms}^2 = \int_{-\infty}^{\infty}(h_2 z^2)^2 F_e(z)dz = 3h_2^2\sigma_z^4 \tag{7}$$

and the chirp profile becomes

$$\Delta p(z) = \frac{1}{\sqrt{3}}\Delta p_{\rm rms}\frac{z^2}{\sigma_z^2}\,. \tag{8}$$

Finally, for a cubic chirp of the form $\Delta\eta(z) = h_3 z^3$ we have

$$\Delta\eta_{\rm rms}^2 = \int_{-\infty}^{\infty}(h_3 z^3)^2 F_e(z)dz = 15h_3^2\sigma_z^6 \tag{9}$$

and the scaled chirp profile is

$$\Delta p(z) = \frac{1}{\sqrt{15}}\Delta p_{\rm rms}\frac{z^3}{\sigma_z^3}\,. \tag{10}$$

For all three cases, the relative cooling rate for a given $\Delta\eta_{\rm rms}$ is defined by

$$\eta_{\rm chirp} = \int_{-\infty}^{\infty} F_e(z)\frac{1}{I_0}I\left(q_h, q_{e,1}, \Delta p(z)\right)dz\,, \tag{11}$$

where $I_0 = \int_{-\infty}^{\infty} F_e(z) I\left(q_h, q_{e,1}, 0\right)dz = I\left(q_h, q_{e,1}, 0\right)$. Here, we have used the fact that the cooling rate $1/N_c$ in a system without amplification is proportional to the local current (see Eqs. (1) and (2) for the $M = 0$ case). The plot of $\eta_{\rm chirp}$ for $q_h = q_{e,1} = 0.6$ is shown in Fig. 3.

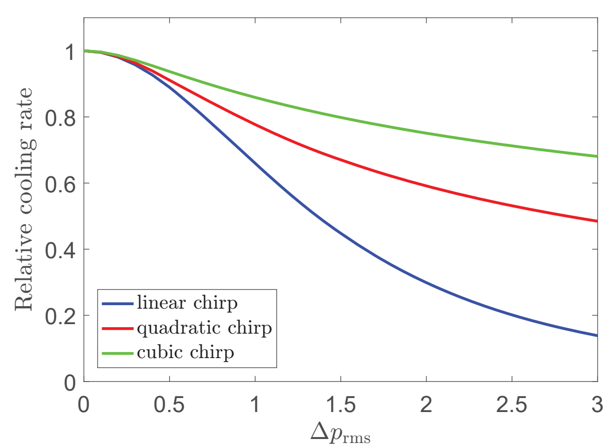

Figure 3: Relative cooling rate versus $\Delta p_{\rm rms}$ for three different chirp profiles (no plasma stages).

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ADDING PLASMA STAGES

When plasma stages are included, the amplification factor for a single stage is modified according to

$$g_j(\varkappa) \to g_j(\varkappa) e^{-i\varkappa q_{e,j}\Delta p}\,, \tag{12}$$

where $q_{e,j}$ is the scaled chicane strength of the plasma section (see Eq. (25) in [3]). For a single plasma stage, the total multiplication factor for the impedance is

$$e^{-i\varkappa(q_{e,1}+q_{e,2})\Delta p}\,, \tag{13}$$

which can be made equal to unity by selecting $q_{e,1} = -q_{e,2}$ (in fact, this case corresponds to the optimum solution without chirp). Thus, it appears that the single-stage case can be made insensitive to e-beam chirp by making the sum of the electron chicane strengths equal to zero.

Assuming two plasma stages with $q_h = q_{e,1} = q_{e,2} = q_{e,3} = q > 0$, the total phase factor that multiplies the impedance is

$$e^{-i\varkappa(q_{e,1}+q_{e,2}+q_{e,3})\Delta p} = e^{-3i\varkappa q\Delta p}\,. \tag{14}$$

Following the treatment of [3], we can conclude that the phase term of Eq. (14) introduces a multiplicative factor $\cos(3\varkappa q\Delta p)$ in the integrand of the $\varkappa$-integral. Thus, in view of Eqs. (1) and (2) for $M = 2$, the modified cooling integral becomes

$$I(q,\Delta p, I_e) = \frac{2q^4}{r_p}\int_0^\infty d\varkappa\,\varkappa^3 e^{-2\varkappa^2 q^2} H^2(\varkappa)H(r_p\varkappa) \times \sin^2(l(I_e)\sqrt{\frac{2\varkappa H(r_p\varkappa)}{r_p}})\cos(3\varkappa q\Delta p)\,, \tag{15}$$

where we have emphasized the dependence of the dimensionless parameter $l \equiv r_p\Omega_p L_d/c$ on the electron current I_e (in fact $l \propto I_e^{1/2} \propto F_e^{1/2}$). In view of this change, we redefine the relative cooling rate according to

$$\eta_{\rm chirp} = \frac{\int_{-\infty}^{\infty} F_e^2(z) I\,(q,\Delta p(z), I_e(z))\,dz}{\int_{-\infty}^{\infty} F_e^2(z) I\,(q,0,I_e(z))\,dz}\,, \tag{16}$$

where we have taken into account the fact that the pre-factor of the cooling rate is now $\propto I_e^2$ (according to Eq. (1)). The new tolerances are plotted in Fig. 4. For this plot, we have assumed $l_{\rm max} = 1.0$ and $q = 0.3$ for a squeeze factor $r_p = 0.2$ (as in [3]). As before, the losses are less severe for quadratic and cubic chirp than they are for linear chirp.

Finally, we consider an alternative configuration with $q_h = q_{e,1} = q > 0$ and $q_{e,2} = q_{e,3} = -q < 0$. In the absence of chirp, this would yield the same cooling time as before. The new impedance phase factor is $e^{-i\varkappa(q_{e,1}+q_{e,2}+q_{e,3})\Delta p} = e^{i\varkappa q\Delta p}$ and the modified cooling integral becomes

$$I(q,\Delta p, I_e) = \frac{2q^4}{r_p}\int_0^\infty d\varkappa\,\varkappa^3 e^{-2\varkappa^2 q^2} H^2(\varkappa)H(r_p\varkappa) \times \sin^2(l(I_e)\sqrt{\frac{2\varkappa H(r_p\varkappa)}{r_p}})\cos(\varkappa q\Delta p)\,. \tag{17}$$

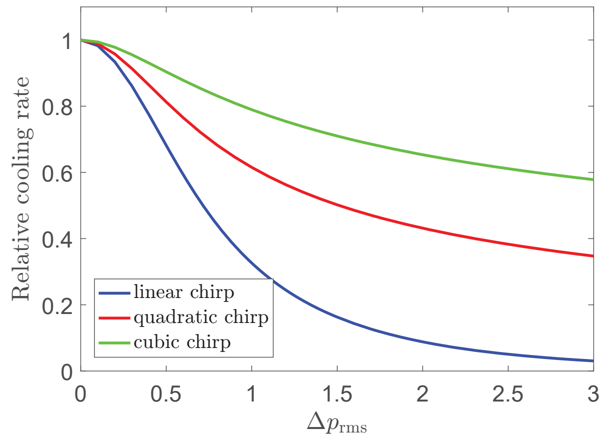

Figure 4: Relative cooling rate versus $\Delta p_{\rm rms}$ for the three different chirp profiles (two plasma stages, case I with all chicane strengths positive).

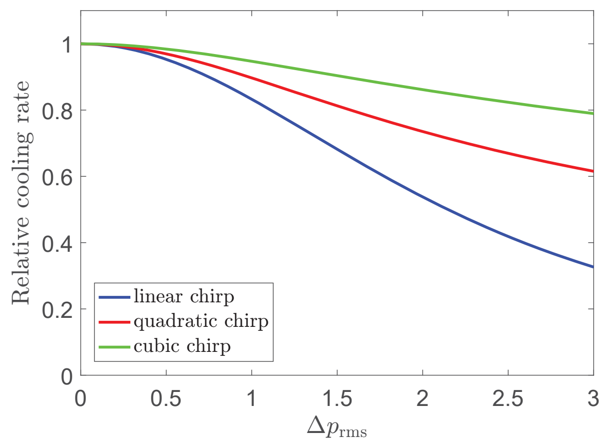

Figure 5: Relative cooling rate versus $\Delta p_{\rm rms}$ for the three different chirp profiles (two plasma stages, case II with two negative chicane strengths).

The relative rates for this case are plotted in Fig. 5 (for the same q, $l_{\rm max}$). This configuration has better tolerances than the previous case with two plasma stages, the cost being that some of the electron chicane strengths have to be negative.

CONCLUSIONS

We have studied the sensitivity of MBEC performance with respect to a non-zero average value of the electron energy variable. In particular, we have incorporated the average energy deviation effect into our frequency-domain formalism, a manipulation that allows us to determine the cooling timescale in the presence of electron beam chirp. From our numerical study, we establish that, even though the chirp does result in a loss of cooling rate, this reduction in performance can be mitigated by minimizing the sum of the electron chicane strengths (in an absolute value sense). This conclusion is in line with a similar observation in [1].

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