

# REACHING LOW EMITTANCE IN SYNCHROTRON LIGHT SOURCES BY USING COMPLEX BENDS\*

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## Abstract

All modern projects of low-emittance synchrotrons follow Multi-Bend Achromat approach. The low emittance is realized by arranging small horizontal beta-function and dispersion in the bending magnets, the number of which varies from 4 to 9 magnets per cell. We propose an alternative way to reach low emittance by use of a lattice element that we call “Complex Bend”, instead of regular dipole magnets. The Complex Bend is a new concept of bending magnet consisting of a number of dipole poles interleaved with strong alternate focusing so as to maintain the beta-function and dispersion oscillating at very low values. The details of Complex Bend, considerations regarding the choice of optimal parameters, thoughts for its practical realization and use in low-emittance lattices, are discussed.

## INTRODUCTION

Modern synchrotron lattice sources are competing intensively to increase X-ray brightness and, eventually, approach the diffraction limit, which sets the final goal of lattice emittance. The trend of minimizing the emittance in modern synchrotrons translates into the reduction of dispersion and beta-functions in their lattice dipoles. Most recent facility upgrades employ MBA lattices [1-5], i.e., introduce a number of dipoles with strong focusing quadrupoles interspersed, which helps maintain lattice functions at smaller values, compared with conventional (Double-Bend) DBA or (Triple-Bend) TBA lattice solutions. The number of dipoles per machine period may vary between 5 (MAX-4) to 19 (MAX-4 upgrade) for the latest designs [6].

In this paper we propose another optics solution to reach ~20 pm-rad low emittance, using a lattice element that we named “Complex Bend” [7, 8] and its evolution “Complex Bend II” [9, 10], which reduces its overall length, lower the quadrupole strength and, therefore, to free up more space in the storage ring lattice available for installing lattice magnets, diagnostics and Insertion Devices. With these solutions we designed the Complex Bend element, which is about the same length as conventional dipole at a lower gradient (we constrained ourselves to stay at or below 250 T/m and the bore diameter of not less than 1 cm). Then we applied the developed solution to a model ring lattice with 18.7 pm-rad emittance, taking NSLS-II ring geometry as the reference.

\*This manuscript has been authored by Brookhaven Science Associates, LLC under Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy

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## COMPLEX BEND

It is customary to relate ring emittance with the number of ring dipoles as:

$$\varepsilon_x = F \frac{E^2}{J_x N_d^3} \quad (1)$$

where  $F$  is a function of ring lattice,  $E$  is the beam energy,  $J_x$  is horizontal partition number and  $N_d$  is the number of dipoles in the ring. In MBA lattice, many facilities increase  $N_d$  and distribute dipoles along arc.

We propose to substantially decrease the beam emittance by converting number of dipoles with the same field polarity,  $N_p$ , into “one element” and keep the bending element,  $N_d$ , the same as 3<sup>rd</sup> generation synchrotron light source ring. Overall, the total number of dipole poles becomes ( $N_d N_p$ ). This will preserve substantial room for insertion devices and associated lattice elements.

Figure 1 shows the concept of the Complex Bend. It is a bending element consisting of dipole poles, interleaved with strong focusing and defocusing quadrupole poles.

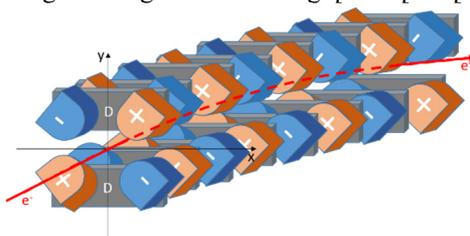


Figure 1: Cartoon illustrating the concept of Complex Bend.

The element provides distributed focusing and bending of the particle beam (Fig. 2). A single cell is featured by the betatron phase advance, which depends on the combination of field gradients in quadrupole and field in the dipole magnets, their lengths and the distance between the consecutive poles.

We approximated the beta function  $\beta_x$  and dispersion  $\eta$  for one cell of Complex Bend with the following expression:

$$\begin{bmatrix} \beta_x(s) \\ \eta(s) \end{bmatrix} \approx \begin{bmatrix} \bar{\beta}_x \\ \bar{\eta} \end{bmatrix} + \begin{bmatrix} \Delta\beta_x \\ \Delta\eta \end{bmatrix} \cdot \cos(k_{CB}s), \quad (2)$$

which is quite accurate for the relatively small phase advances that we consider in our cases of interest. Here  $L_{CB} = 2(L_Q + L_B + 2L_D)$  is the length of the cell,

$L_Q, L_B, L_D$  are the lengths of the Complex Bend quadrupole, dipole, and drift, respectively, and  $k_{CB} = \frac{2\pi}{L_{CB}}$ . We used these expressions to find synchrotron radiation integrals for a given ring lattice and to obtain analytic dependencies of H-function,  $H = \gamma_x \eta^2 + 2\alpha_x \eta \eta' + \beta_x \eta'^2$ , on the dipole and quadrupole fields. We worked out analytic expressions of  $\beta_x, \Delta\beta_x$  and  $\eta, \Delta\eta$  for the case when  $K_{1F} = -K_{1D}$ .

Using these expressions, we calculate emittance of the ring with the Complex Bends as:

$$\varepsilon_x \approx C_q \gamma^2 \frac{\bar{\eta}^2}{R_B \beta_x}, \quad (3)$$

where  $C_q = 3.832 \cdot 10^{-13} \text{ m}$ , and  $R_B$  is beam bending radius in the dipole.

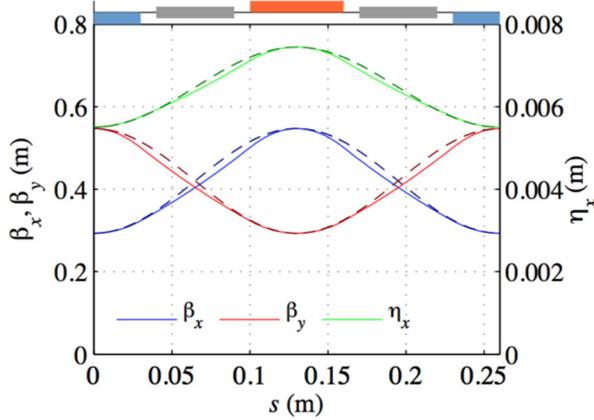


Figure 2: Single cell of Complex Bend and Twiss functions. The solid lines represent computer simulations; the dashed lines represent the approximate analytical solutions.

As follows from the above, the Complex Bend results in smaller beta-functions and dispersion, thus, resulting in smaller integral of H-function and emittance. The required range of gradients  $K_l$  is at the level of  $50 \text{ m}^{-2}$ . With Complex Bend lattice, we found that the emittance of  $15 \text{ pm rad}$  is feasible for a ring of the NSLS-II [11] circumference (792 m) with 120 complex bend and gradients at  $\sim 500 \text{ T/m}$  for the beam energy of 3 GeV. To produce such high gradients field, one choice is based on superconducting magnet technology and thus is far more complex than regular electromagnets.

## COMPLEX BEND II

We have considered a modification of the Complex Bend element, aiming to reduce its overall length, lower the quadrupole strength and, therefore, to free up more space in the storage ring lattice available for installing lattice magnets, diagnostics and Insertion Devices. We realized this modification by removing dipole poles from the element and enabling the bending either by quadrupole poles, shifted transversely, or by external field realized by an electromagnet with the quadrupole poles fitted inside the gap.

CBII [9,10] consists of several pairs of quadrupoles, every pair representing a single cell from the sequence of repetitive cells, with the “square wave” field distribution along the s-axis. The bending angle is realized by shifting both quadrupoles off-center relative to the beam orbit. A single cell of the CBII contains a focusing quadrupole (QF), a drift, a defocusing quadrupole (QD), and another drift. In Fig. 3 we illustrate the principle of 3 consecutive poles or 1.5 cells of the CBII featuring the geometry of the beam orbit that consists of a sequence of arcs with variable radius separated by short drifts. The total angle per single cell is distributed between the two quadrupoles according to the parameter  $0 < \alpha < 1$ . We define the entrance angles as:

$$\Delta xp_F = \frac{\alpha_d}{2} \alpha, \quad \Delta xp_D = \frac{\alpha_d}{2} (1 - \alpha),$$

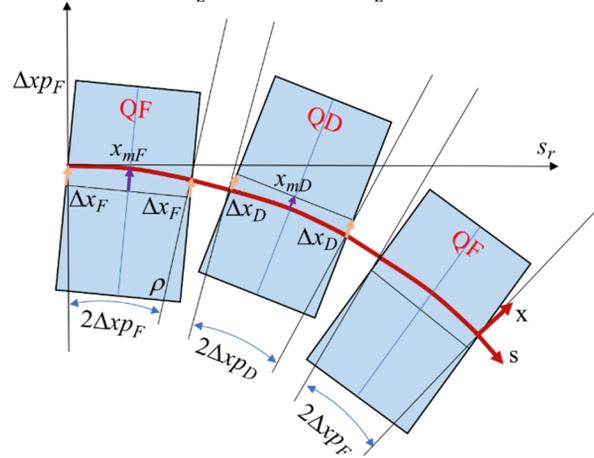


Figure 3: Sketch of CBII geometry.

In our analysis we will be using the following parameters (Table 1), which are relevant to the existing layout of NSLS-II storage ring.

Table 1: Parameters Used in the Analysis of Complex Bend II

Number of CBII elements in the ring	60
Number of cells per CBII element	5
Total number of cells in the ring	300
Total number of poles in the ring	600
Angle per cell, mrad	20.94
Angle per CBII element, mrad	105
Quadrupole pole length, m	0.28
Drift length, m	0.03
Cell length, m	0.62
Complex Bend element length, m	3.1
Beam energy, GeV / unitless	3 / 5871
Magnetic rigidity, T·m	10.0
Scaled gradient in CBII quadrupoles, $\text{m}^{-2}$	25 / -25
Corresponding field gradient, T/m	250, -250
Fraction of QF angle in the cell's angle	0.35

## STABILITY CONSTRAINT FOR THE RING OPTICS BASED ON COMPLEX BEND II

$I_2, I_3, I_4$  and  $I_5$  are the radiation integrals around the ring. Recall that the partition numbers and synchrotron integrals are written as [12]:

$$J_x = 1 - \frac{I_4}{I_2}, J_z = 2 + \frac{I_4}{I_2}, I_2 = \oint \frac{ds}{\rho^2}, I_3 = \oint \frac{ds}{\rho^3}, I_4 = \oint \frac{\eta}{\rho} \left( \frac{1}{\rho^2} + 2K_1 \right) ds, I_5 = \oint \frac{H_x}{|\rho|^3} ds$$

The radiation integrals for CBII can be expressed as:

$$I_2 = 2\pi \left( \frac{\alpha}{\rho_{Fav}} + \frac{(1-\alpha)}{\rho_{Dav}} \right)$$

$$I_3 = 2\pi \left( \frac{\alpha}{\rho_{Fav}^2} + \frac{(1-\alpha)}{\rho_{Dav}^2} \right)$$

$$I_4 \approx \frac{N_p \cdot 2\eta_{Fav} \cdot K_{1F} \cdot L_Q}{\rho_{Fav}} - \frac{N_p \cdot 2\eta_{Dav} \cdot K_{1D} \cdot L_Q}{\rho_{Dav}} \quad \text{where}$$

we neglected the integral contributions from  $\frac{\eta}{\rho^3}$ , since they are much smaller than those terms containing  $K_1$ .

$$I_5 \approx 2\pi \left( \frac{\alpha}{\rho_{Fav}^2} \frac{\eta_{Fav}^2}{\beta_{Fav}} + \frac{(1-\alpha)}{\rho_{Dav}^2} \frac{\eta_{Dav}^2}{\beta_{Dav}} \right)$$

Storage ring emittance and energy spread are:

$$\varepsilon_x = C_q \gamma^2 \frac{I_5}{I_2 - I_4} = C_q \cdot \gamma^2 \cdot \frac{I_5}{J_x \cdot I_2}$$

$$\frac{dE}{E} = \gamma \sqrt{C_q \cdot \frac{I_3}{2I_2 + I_4}}$$

where  $C_q \approx 3.84 \cdot 10^{-13} m$ .

$J_x$  is determined by  $I_2$  and  $I_4$ , and, in turn,  $I_2$  depends only on the dipole bending radius. To maintain the longitudinal or horizontal stability of the beam dynamics in the ring we need to ensure that  $J$  elements of the damping distribution are always positive. The Complex Bend optics allow us to simplify the synchrotron integrals presented above. For a sequence of  $N_F$  focusing and  $N_D$  defocusing shifted quadrupole magnets, the 4<sup>th</sup> synchrotron integral  $I_4$  can be approximated as:

$$I_4 = \int \frac{2\eta_F K_{1F}}{\rho_F} ds + \int \frac{2\eta_D K_{1D}}{\rho_D} ds$$

The change in  $I_4$  due to CBII poles or combined function magnets can be estimated as:

$$I_4 \approx 2N_F L_F \eta_{Fav} K_{1F}^2 \Delta x_F + 2N_D L_D \eta_{Dav} K_{1D}^2 \Delta x_D$$

For a periodic structure,  $N_F = N_D = N_Q, L_F = L_D = L_Q$ . In the particular case when there is no net change in  $I_4$  due to CBII poles or combined function magnets, we determine that the relationship between the translations and gradients of the focusing and defocusing poles is:

$$\eta_{Fav} K_{1F}^2 \Delta x_F + \eta_{Dav} K_{1D}^2 \Delta x_D = 0 \quad (4)$$

This constraint maintains the ring optics with Complex Bends II stable in all three planes by preserving damping partitions positive. We note that this constraint works for any ring optics, combined function magnets or shifted

quadrupoles, i.e. CBII elements. In the actual lattice design, one may adjust this constraint to reach minimum emittance by increasing  $J_x$ , which should maintain damping partitions always positive.

## PARAMETERS OF COMPLEX BEND II FOR AN LATTICE OPTION OF THE NSLS-II UPGRADE

Using the formalism described above, we designed a storage ring lattice with Complex Bend II elements in the place of the conventional dipoles of Double Bend Achromat lattice. Using the CBII transport matrices, we calculated matched solutions for Twiss functions and dispersion. Then we estimated the ring parameters and optimized it yielding the stability constraint.

In our design we used the parameters of CBII element from Table 1. The shifts of QF and QD poles are 1.13 mm and -1.79 mm correspondingly. The latter shift is large as compared with the total diameter of the aperture inside the poles of 10 mm and we assessed a solution for the magnet design, which enables us to reduce this shift.

In Table 2 we presented the ring parameters using the expressions from Table 1 and confirmed by the lattice code Elegant [13].

Table 2: Preliminary parameters of a storage ring based on Complex Bend II. Horizontal emittance is 18.7 pm-rad at 3 GeV.

Circumference, m	792
Emittance, pm-rad	18.7
Energy spread, %	0.09
Momentum compaction	3.8e-5
Natural chromaticity	-199/-260
Tunes	97.82/112.52
Beta-functions in straights, m	4.1/2.5
Ratio of straights to circumference	28.6%
Synchrotron radiation power, keV/turn	346

The lattice functions are presented in Fig. 4. Arrangement of the quadrupoles looks similar to NSLS-II lattices, however the CBII lattice features emittance that is factor of 100 smaller than that for the NSLS-II DBA lattice.

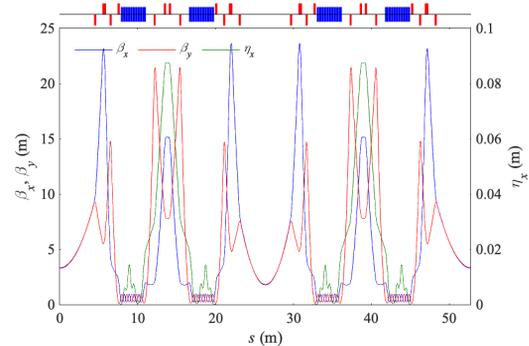


Figure 4: NSLS-II upgrade based on CBII design.

The quadrupole gradient is reduced to  $\sim 250$  T/m in complex bend II as compared to  $\sim 500$  T/m in complex bend. Even the reduce gradient is quite high but is achievable

with a small beam aperture of  $\sim 10$  mm with Halbach permanent magnet quadrupole (PMQ) [14]. We are presently building a Complex Bend II prototype for future experiments at Brookhaven Lab's Accelerator Test Facility (ATF), evaluated at a beam energy of 50 MeV. The CBII prototype will be 46 mm long and consists of five cells, with the field gradient set at 150 T/m. We scaled the Complex Bend parameters from 3 GeV down to 50 MeV.

## SUMMARY

In this paper we presented the conceptual details of Complex Bend element. An electron ring built from such elements features in low emittance, while preserving substantial room for insertion devices and associated lattice elements. Then we worked out NSLS-II upgrade lattice solutions, where the original NSLS-II dipoles were replaced with the elements of the Complex Bend II to get 18.7 pmrad beam emittance. The required quadrupole gradients of 250 T/m can be achieved by using the permanent magnet technology. A prototype of the Complex Bend is being developed for magnetic measurements and will test with beam at BNL's ATF at 50 MeV.

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