

# MAXIMIZING 2-D BEAM BRIGHTNESS USING THE ROUND TO FLAT BEAM TRANSFORMATION IN THE ULTRALOW CHARGE REGIME\*

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## Abstract

A method for maximizing 2-D beam brightness in an RF photoinjector in an ultralow charge regime (<1 pC) is presented. Theory and particle tracking simulations suggest that by utilizing the flat beam transform technique, normalized projected emittances smaller than 5 nm in one spatial dimension can be obtained at the UCLA Pegasus facility with up to 100 fC beam charge. This is achieved by starting the beam at the photocathode in a tunable magnetic field in order to introduce canonical angular momentum into the beam. A skew quadrupole triplet is then used to block-diagonalize the beam matrix and recover the vastly different eigenemittances as the projected emittances. Advanced emittance measurement techniques, including 4-D grid-based and two-quadrupole scan routines, are used for the reconstruction of the 4-D beam matrix. Preliminary measurements indicate a small emittance smaller than the thermal emittance of an equivalent round beam.

## INTRODUCTION

A common metric for overall beam quality is the beam brightness, defined as

$$B_{4D} = \frac{Q}{\epsilon_{4D}}, \quad (1)$$

where  $Q$  is the beam charge and  $\epsilon_{4D}$  is the 4-D emittance. The 4-D beam brightness is effectively the number of electrons in the 4-D phase space volume of the beam associated with the two transverse spatial dimensions. One can define a similar metric for one spatial dimension:

$$B_{2D} = \frac{Q}{\epsilon_{2D}}, \quad (2)$$

where  $\epsilon_{2D}$  is the 2-D projected emittance in the desired spatial dimension. Maximizing 2-D beam brightness has broad applications to scenarios where a high-quality beam is only required in one spatial dimension, but there are less stringent requirements on the beam quality in the other. These applications include slab-geometry dielectric laser acceleration, where the phase space acceptance is extremely small in the small gap, but orders of magnitude larger in the other dimension. Another example of such an application is electron imaging of certain sample geometries where high resolution imaging is only desired in one spatial dimension.

There is also a fundamental reason to minimize the 2-D emittance that is related to the non-squeezing theorem from

Gromov [1]. The theorem states that it is impossible to find a symplectic transformation that reshapes a finite 4-D spherical volume into an infinitely large cylindrical volume of smaller (phase-space) radius than the initial sphere. In the language of accelerator and beam physics, this means that there are strong limits on the freedom to reshape the phase space occupied by the beam in a transport system for a given target application. More quantitatively, not only is the phase space volume preserved, as the often cited Liouville theorem implies, but also Hamiltonian transport keeps invariant another set of quantities associated with the beam phase space distribution: the so-called eigenemittances [2]. The lowest of these invariants sets the limit on the smallest projected emittance achievable [3].

The Round to Flat Beam Transformation (FBT) makes use of emittance exchange to shrink the 2-D projected emittance in one spatial dimension at the expense of the other (while maintaining constant the 4-D phase space volume of the beam). The scheme is based on generating the beam in a region with non-zero magnetic field, which imparts canonical angular momentum on the beam and makes one eigenemittance much smaller than the other. The FBT transformation simply retrieves the eigenemittance as the projected emittance. The beam matrix of a round magnetized beam can be written as follows:

$$\sigma^{4D} = \begin{pmatrix} \sigma_c & 0 & 0 & \mathcal{L} \\ 0 & \sigma_{x'x'} & -\mathcal{L} & 0 \\ 0 & -\mathcal{L} & \sigma_c & 0 \\ \mathcal{L} & 0 & 0 & \sigma_{y'y'} \end{pmatrix}, \quad (3)$$

where  $\sigma_c$  is the beam size on the cathode,  $\sigma_{x'x'} = \sigma_{y'y'}$  represent the initial angles in the beam and

$$\mathcal{L} = \frac{eB_c}{2\gamma\beta mc} \sigma_c^2 = \frac{L}{\gamma\beta} \quad (4)$$

is proportional to the angular momentum in the beam, where  $B_c$  is the magnetic field on the cathode, and  $\gamma$  and  $\beta$  are the relativistic factors. The beam eigenemittances can be found by [4]

$$\epsilon_1 = \frac{1}{2} \sqrt{-\text{Tr}[(\sigma^{4D} J)^2] + \sqrt{\text{Tr}[(\sigma^{4D} J)^2] - 16 \det(\sigma^{4D})}}, \quad (5)$$

$$\epsilon_2 = \frac{1}{2} \sqrt{-\text{Tr}[(\sigma^{4D} J)^2] - \sqrt{\text{Tr}[(\sigma^{4D} J)^2] - 16 \det(\sigma^{4D})}}, \quad (6)$$

where  $J$  is the symplectic matrix.

Calculating these quantities for the magnetized beam matrix in Eq. (3) it can be shown [5] that given a large enough

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$L$ , we have  $\epsilon_+ \gg \epsilon_-$  where  $\epsilon_{\pm}$  is the greater (smaller) of the two eigenemittances. Formally,

$$\epsilon_{\pm} = \sqrt{\epsilon_u^2 + L^2} \pm L, \quad \epsilon_u = \sqrt{\frac{MTE}{mc^2} \sigma_c^2}, \quad (7)$$

where  $\epsilon_u$  is the thermal emittance of the cathode which depends on the Mean Transverse Energy (MTE) in the photoemission process. In the limit of large  $L$  (i.e. angular momentum dominated beam),

$$\epsilon_- \approx \frac{MTE}{eB_c c}, \quad \epsilon_+ \approx 2L + \epsilon_-. \quad (8)$$

This dependence is very attractive given the recent progress in achieving low MTE from advanced photocathodes [6] and provides an interesting alternative to defeat the brightness scaling in a photoinjector [7].

Retrieving the eigenemittances as normalized projected emittances is accomplished using three skew quadrupoles as proposed by Derbenev in 1998 [8] in the framework of electron cooling. In 2001, Brinkmann et al. extended the idea of the FBT for colliders [9]. In 2003, Kim produced a cohesive summary of the theory of the FBT [5], much of which was originally derived in [10] and [11]. In 2006, the first experimental demonstration of FBT was shown to produce an emittance ratio of 100, with a small emittance smaller than the thermal emittance of an equivalent round beam [12]. A more detailed account of this work can be found in [13, 14]. A relatively simple analytical solution of the FBT is given in [15]. Whereas past publications have focused on a large emittance ratios and relatively large beam charge applications, this paper focuses mainly on reducing the small projected emittance as much as possible in the ultra-low charge regime where space charge effects are negligible.

## EXPERIMENTAL SETUP

This experiment was performed at the Pegasus laboratory at the University of California, Los Angeles (UCLA) [16]. A 35 mm × 35 mm × 17.4 mm neodymium (N38) magnet is mounted on an actuator with 10 cm of longitudinal range of motion behind a 3.5 mm thick photocathode inside a 1.6 cell radiofrequency photoinjector (2.856 GHz) [17]. The maximum magnetic field at the front of the cathode is approximately 0.3375 T. The dependence of the field on the distance between the magnet and the photocathode is shown in Fig. 1. The photocathode is illuminated with a 100-fs rms pulse length laser with a pulse energy that can be varied (between 0–10 μJ in this experiment) in order to adjust the electron beam charge. The beam had a total energy at the exit of the gun of approximately 3.65 MeV and was focused by a solenoid to produce a beam of minimum size at a skew quadrupole triplet, which is located approximately 2.8 m from the photocathode. The skew quadrupoles used for the flat beam adapter have an effective length of 10.5 cm, a gradient of approximately 0.484  $\frac{T}{Am}$  and are followed by normal quadrupole triplet. These normal quadrupoles have

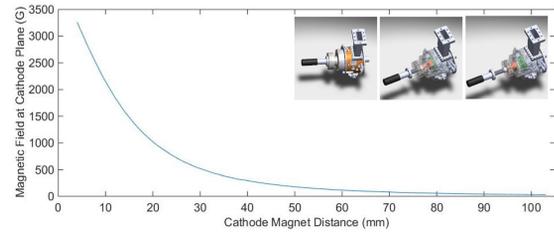


Figure 1: Magnetic field on the front face of the cathode as a function of distance from the front face of the neodymium magnet. Inset (left to right): gun with magnet actuator assembly, schematic of the magnet up against the back of the cathode (green), schematic of the magnet positioned far from the cathode (green).

an effective length of 7.68 cm and a gradient of approximately 0.45  $\frac{T}{Am}$ . TEM grids are mounted on a removable stage 3.846 m downstream from the cathode. The beam was measured on a YAG screen 4.545 m from the cathode using two-quadrupole scan [18] and grid shadow [19] 4-D beam matrix reconstruction techniques. See Fig. 2 for a schematic of the Pegasus beam line.

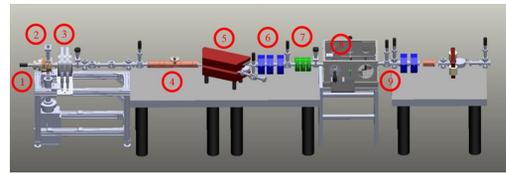


Figure 2: Schematic of the Pegasus beam line at UCLA. List of beam line elements: 1) permanent magnet to introduce a magnetic field on the cathode, 2) 1.6 cell RF gun, 3) focusing solenoid, 4) booster LINAC (not used in this experiment), 5) dipole spectrometer, 6) flat beam adapter comprised of a skew quadrupole triplet, 7) normal quadrupole triplet for focusing, 8) TEM grids for emittance measurement, 9) diagnostic screens.

## SIMULATION AND OPTIMIZATION

The FBT beamline was simulated using the General Particle Tracer (GPT) [20] with a gradient descent minimization to optimize the skew quadrupole currents (i.e. either minimizing the emittance ratio or minimizing  $P_2$ ):

$$P_2 = \frac{\epsilon_x \epsilon_y}{\epsilon_{4D}}. \quad (9)$$

This quantity is unity when the beam matrix is completely uncoupled, as the off-diagonal elements of  $\epsilon_{4D}$  are zero. In general, the product of projections of a hypervolume is greater than or equal to the hypervolume, so  $P_2$  is always greater than or equal to unity, as the emittances are proportional to the area in phase space that the beam occupies. Simulations produced solutions for the currents in the skew quadrupoles necessary to minimize one projected emittance.

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The smallest emittance attainable in particle tracking simulations is shown as a function of beam charge in Fig. 3. In

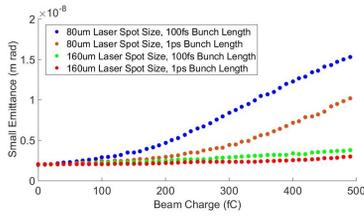


Figure 3: Small emittance as a function of beam charge and bunch length from particle tracking simulation.

the actual experiment, the skew quadrupoles were optimized to obtain a very narrow up-right aspect ratio for the beam on the final screen after verifying that the beam was nearly focused on a screen immediately following the flat beam adapter. Machine learning optimization has also been considered for this task. In collaboration with SLAC, a neural network model of the Pegasus beam line was recently trained on measured data and used to predict optimal values for the skew quadrupoles. Further study is needed to assess rigorously the performance of this and other machine learning based methods.

## RESULTS AND DISCUSSION

A simple verification for the flat beam adapter was qualitatively carried out using the method of the three screens. Quantitative measurements were hindered by the large point spread functions of the screens used for this measurement. The scaling with the initial angular momentum was verified changing the laser spot size on the cathode (see Fig. 4). As can be seen from Eqs. (7) and (8), the large emittance increases as the square of the initial laser spot size on the photocathode. Consequently, for constant beta functions at the screen, the large electron spot size varies approximately linearly with the laser spot size, since

$$\sigma_{lg,screen} = \sqrt{\epsilon_+ \beta}, \quad (10)$$

while the small electron beam spot size should stay constant as a function of the initial laser spot. The emittance

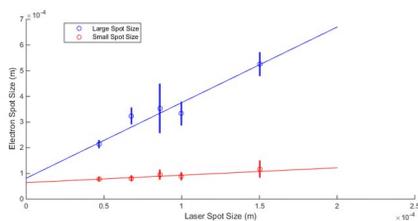


Figure 4: Flat beam rms spot sizes as a function of laser rms radius on the photocathode.

tance of the beam was measured using two techniques: the two-quadrupole scan [18] and the TEM grid method [19].

The two-quadrupole scan is based on varying the normal quadrupoles after the flat beam adapter and measuring the corresponding second order moments of the beam distribution at the detector:

$$\sigma_{xx} = \sigma_x^2, \quad \sigma_{yy} = \sigma_y^2, \quad (11)$$

$$\sigma_{xy} = \frac{-2\sigma_{x,45}^2 + \sigma_x^2 + \sigma_y^2}{2} = \frac{2\sigma_{y,45}^2 - \sigma_x^2 - \sigma_y^2}{2}. \quad (12)$$

After a sufficient number of measurements, it is possible to fit all ten unique beam matrix elements for the flat beam. The two-quadrupole scan technique is a multishot measurement which suffers from machine fluctuations and has a limited resolution on the small emittance value. The large emittance was found in very good agreement with the predictions (see Fig. 5). In order to resolve the small emittance,

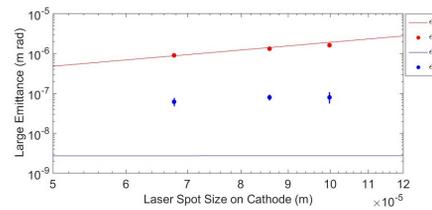


Figure 5: Flat beam emittances as measured by a quadrupole scan as a function of laser spot size on the photocathode.

the TEM grid [19] (see Fig. 6) reconstruction technique was employed. Because the grids are equally spaced in the  $x$  and  $y$  directions, a flat beam cannot benefit from the 4-D nature of this reconstruction technique, as the angles in the beam in the large emittance direction make it impossible to resolve the corresponding grid bars. However, a 2-D reconstruction can be used for the small projected emittance. The emittance

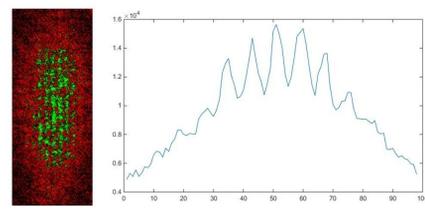


Figure 6: Left: Flat beam passing through TEM grids. Right: Projection of the flat beam image along the  $x$  axis.

measured using TEM is still larger than theory predicts by an order of magnitude. An erroneous quadrupole moment in the gun is being investigated as a possible cause of the inability to minimize the small emittance to the theoretical levels [21]. If this is found to be the case, simulations suggest that a quadrupole coupling corrector [22, 23] could be used to retrieve the smallest eigenemittance as the normalized projected emittance. Other effects such as chromatic aberrations in the quadrupoles or a systematic error in the emittance measurements are being investigated as well.

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## REFERENCES

- [1] M. Gromov, "Pseudo holomorphic curves in symplectic manifolds," *Inventiones Mathematicae*, vol. 82, p. 307, 1985.
- [2] A. J. Dragt, F. Neri, and G. Rangarajan, "General moment invariants for linear hamiltonian systems," *Phys. Rev. A*, vol. 45, pp. 2572–2585, Feb 1992.
- [3] L. D. Duffy and A. J. Dragt, "Utilizing the eigen-emittance concept for bright electron beams," in *Advances in Imaging and Electron Physics*, vol. 193, pp. 1–44, Elsevier, 2016.
- [4] C. Xiao, O. Kester, L. Groening, H. Leibrock, M. Maier, and P. Rottländer, "Single-knob beam line for transverse emittance partitioning," *Physical Review Special Topics-Accelerators and Beams*, vol. 16, no. 4, p. 044201, 2013.
- [5] K.-J. Kim, "Round-to-flat transformation of angular-momentum-dominated beams," *Physical Review Special Topics-Accelerators and Beams*, vol. 6, no. 10, p. 104002, 2003.
- [6] L. Cultrera, S. Karkare, H. Lee, X. Liu, I. Bazarov, and B. Dunham, "Cold electron beams from cryocooled, alkali antimonide photocathodes," *Phys. Rev. ST Accel. Beams*, vol. 18, p. 113401, Nov 2015.
- [7] I. V. Bazarov, B. M. Dunham, and C. K. Sinclair, "Maximum achievable beam brightness from photoinjectors," *Phys. Rev. Lett.*, vol. 102, p. 104801, Mar 2009.
- [8] Y. Derbenev, "Adapting optics for high energy electron cooling," *University of Michigan, UM-HE-98-04*, 1998.
- [9] R. Brinkmann, Y. Derbenev, and K. Flöttmann, "A low emittance, flat-beam electron source for linear colliders," *Physical Review Special Topics-Accelerators and Beams*, vol. 4, no. 5, p. 053501, 2001.
- [10] A. Burov, S. Nagaitsev, A. Shemyakin, and Y. Derbenev, "Optical principles of beam transport for relativistic electron cooling," *Physical Review Special Topics-Accelerators and Beams*, vol. 3, no. 9, p. 094002, 2000.
- [11] A. Burov, S. Nagaitsev, and Y. Derbenev, "Circular modes, beam adapters, and their applications in beam optics," *Physical Review E*, vol. 66, no. 1, p. 016503, 2002.
- [12] P. Piot, Y.-E. Sun, and K.-J. Kim, "Photoinjector generation of a flat electron beam with transverse emittance ratio of 100," *Physical Review Special Topics-Accelerators and Beams*, vol. 9, no. 3, p. 031001, 2006.
- [13] Y.-E. Sun, "Angular-momentum-dominated electron beams and flat-beam generation," *PhD thesis, University of Chicago*, 2005.
- [14] Y.-E. Sun, P. Piot, K.-J. Kim, N. Barov, S. Lidia, J. Santucci, R. Tikhoplav, and J. Wennerberg, "Generation of angular-momentum-dominated electron beams from a photoinjector," *Physical Review Special Topics-Accelerators and Beams*, vol. 7, no. 12, p. 123501, 2004.
- [15] B. E. Carlsten and K. A. Bishofberger, "Simple algorithm for designing skew-quadrupole cooling configurations," *New Journal of Physics*, vol. 8, no. 11, p. 286, 2006.
- [16] J. Maxson, D. Cesar, G. Calmasini, A. Ody, P. Musumeci, and D. Alesini, "Direct measurement of sub-10 fs relativistic electron beams with ultralow emittance," *Physical review letters*, vol. 118, no. 15, p. 154802, 2017.
- [17] D. Alesini, A. Battisti, M. Ferrario, L. Foggetta, V. Lollo, L. Ficcadenti, V. Pettinacci, S. Custodio, E. Pirez, P. Musumeci, *et al.*, "New technology based on clamping for high gradient radio frequency photogun," *Physical Review Special Topics-Accelerators and Beams*, vol. 18, no. 9, p. 092001, 2015.
- [18] E. Prat and M. Aiba, "Four-dimensional transverse beam matrix measurement using the multiple-quadrupole scan technique," *Physical Review Special Topics-Accelerators and Beams*, vol. 17, no. 5, p. 052801, 2014.
- [19] D. Marx, J. G. Navarro, D. Cesar, J. Maxson, B. Marchetti, R. Assmann, and P. Musumeci, "Single-shot reconstruction of core 4d phase space of high-brightness electron beams using metal grids," *Physical Review Accelerators and Beams*, vol. 21, no. 10, p. 102802, 2018.
- [20] "General particle tracer," <http://www.pulsar.nl/gpt/>.
- [21] P. Denham, F. Cropp, J. Giner Navarro, and P. Musumeci, "A study of quadrupole coupling error correction in rf photoinjectors," in *Proceedings of the North American Particle Accelerator Conference (NAPAC 2019)*, (Lansing, Michigan, USA), September 2019.
- [22] D. H. Dowell, F. Zhou, and J. Schmerge, "Exact cancellation of emittance growth due to coupled transverse dynamics in solenoids and rf couplers," *Phys. Rev. Accel. Beams*, vol. 21, p. 010101, Jan 2018.
- [23] L. Zheng, J. Shao, Y. Du, J. G. Power, E. E. Wisniewski, W. Liu, C. E. Whiteford, M. Conde, S. Doran, C. Jing, C. Tang, and W. Gai, "Experimental demonstration of the correction of coupled-transverse-dynamics aberration in an rf photoinjector," *Phys. Rev. Accel. Beams*, vol. 22, p. 072805, Jul 2019.